Model Selection in Estimating the Certainty Equivalent Discount Rate

Ben Groom (UCL)

Phoebe Koundouri (University of Reading)

Ekaterini Panopoulou and Theo Pantelides (University of Piraeus, Greece)

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Background (1)

- Emergence of a long-term policy arena: e.g. climate change, biodiversity conservation, long-lived infrastructure.
- Conventional discounting gives CBA a 'defective telescopic faculty' (Pigou 1920). A 'tyranny' of current over the future generations (Chichilnisky 1996)
- Declining Discount Rates (DDRs): a path through the 'dark jungles of the second best' (Baumol 1968)?
- DDRs in the Green Book (2003) for long-term projects ➔ how can practitioners define the schedule of DDRs?

Rationales for DDRs (see e.g. Pearce et al 2003)

1) 'Hyperbolic' Discounting (Loewenstein and Prelec 1992, Laibson 1997, etc)
2) Social Choice (e.g. Chichilnisky 1997, Li and Löfgren 2000, 2001): DDRs appeal to certain axioms of social choice and sustainability: non-dictatorship
3) Uncertainty:
   - **Growth**: Gollier (2002). DDRs depend upon risk
   - **Discount Rate**: Weitzman (1998). The 'certainty equivalent' rate is a declining function of time.

Background (2):

Determining the schedule of DDRs

Some theories are easier to implement than others:

- **Gollier**: detailed knowledge of the utility function (risk and precaution preferences: 5th derivatives of utility!!),
- **Social Choice**: requires a number of arbitrary choices concerning social weights
- **Weitzman:** only requires Characterisation of uncertainty of *r*.

Weitzman approach has received the most attention, but with alternative interpretations of uncertainty:

i) **Weitzman (AER 2001):** 'Gamma Discounting': survey of 2000 Economists (with a 'Blue-Ribbon' group of 50) ➔ current uncertainty/disagreement about the rate to use **now**

ii) **Newell and Pizer (JEEM 2003):** We are pretty certain about today's discount rate, but uncertain about the future ➔ Time series analysis: the past is informative about the future. Resulting schedule increases the value of carbon reduction by 84% compared to constant discounting.

Overview

1) Summary
2) The Certainty Equivalent Discount Rate
3) Newell and Pizer's (JEEM 2003) approach
4) Importance of Model Selection
5) Method
6) Data
7) Results (US) and Simulations
8) Model Selection
9) Policy Implications
10) Conclusion

1. Summary

We focus on the N&P (2003) approach to estimating the schedule of DDRs. We make the following points concerning the estimation of the certainty equivalent discount rate:

1) If we believe that the past is informative about the future we would be well advised to characterise the past as realistically as possible. Characterising uncertainty is important for operationalising a theory of DDRs that depends based on uncertainty and which relies upon the forecasting over long-time horizons (400 years +).

2) Specification testing in the case of US and UK interest rate data provides a natural progression towards models that account for changes in the data generating process: structural breaks, regime shifts etc.

3) We compare N&Ps model to the well specified alternatives and compare the outcome of CBA for climate change (US) and nuclear power (UK) in order to illustrate the importance of model selection.

4) Model selection can have important policy implications for the long-term policy arena

2. The Certainty Equivalent Discount Rate

Under certainty the discount factor at time t, P_t, is:

$$P_t = \exp\left(-\sum_{i=1}^{t} r_i\right)$$

When $\boldsymbol{r}$ is stochastic then the Certainty Equivalent Discount factor for a **risk neutral** agent becomes:

$$E[P_t] = E\left[\exp\left(-\sum_{i=1}^{t} r_i\right)\right]$$

The Certainty Equivalent Discount **Rate**, $r_w(t)$, is defined by:

$$-\frac{dE(P_t)/dt}{E(P_t)} = r_w(t)$$

Weitzman and N&P: With sufficient persistence this is a declining function of time. With the gamma discounting this is given, with N&P this is an empirical question.

3. Newell and Pizer's Approach

AR(3) model of the time series process:

$$\begin{aligned} r_t &= \eta + e_t \\ e_t &= \rho^3 e_{t-3} + \rho^2 e_{t-2} + \rho \; e_{t-1} + \xi_t \end{aligned}$$

where $\xi_t \sim N\left(0, \sigma_\xi^2\right)$, $\eta \sim N\left(\eta, \sigma_\eta^2\right)$

Leads to an expression for the certainty equivalent discount rate:

$$r_w(t) = \overline{\eta} - t\sigma_\eta^2 - \sigma_\xi^2 f(\rho, t)$$

- The theoretical model gives DDRs
- Estimated using US interest data. Numerical simulation taking 100000 forecasts over 400 years.
- Cannot distinguish clearly between random walk and mean reverting models
- RW (persistence) leads to a quicker decline in discount rates.

4. The Importance of Model Selection in inference and forecasting

- N&P model characterises uncertainty in a very limited way: the data generating process is time invariant, cannot easily distinguish between MR and RW

- Such characteristics determine the nature of the schedule of DDRs and hence are important for CBA

Model Selection:

- The selected model should be able to capture the dynamics of the data generating process in order to achieve an accurate description of the series.

- Complexity and restrictions of the model should correspond to the data generating process. If not, inference will be misleading and forecasting poor.

- Model selection should be based on data observation and statistical testing (e.g. unit roots, serial correlation, heteroscedasticity and parameter instability)

- An out of sample forecasting exercise can be a useful tool for measuring performance.

5. Method and Models

- Estimate N&P model: a simple AR(3) model

- Tests for persistence (random walk vs mean reversion) heteroscedasticity (LM test for ARCH), parameter stability.

- Apply appropriate further models as required:

  i) AR(p) - (I)GARCH(l,m)

  ii) Regime Switching

  iii) State Space models

- Compare well specified models on the basis of a number of efficiency and forecasting criteria:

  i) Coefficients of variation

  ii) 5%-95% Quantiles of the forecast

  iii) MSE's of sample forecasts

- Simulations undertaken: random draws of parameters and shocks from empirical distribution, then forecast.

6. Data

US nominal interest rates: 10 year bond constant maturity yield

UK nominal Interests Rates: Consol Yield

- Period: 1800-2001

- Adjusted using 10 year moving average inflation measure

- Accounted for the extreme fluctuations in the real rates seen during the 70's

- Continuously compounded equivalents

7. Results (U.S.) and Simulations

- N&P AR(3) model is misspecified in both the US and the UK cases:
    - Test for persistence inconclusive in US case (many tests): estimated both MR and RW models (MR in UK case)
    - Residuals are ARCH (LM test)
- The AR(p)- IGARCH(l,m) is employed in order to account for conditional heteroscedasticity.
    - Persistence leads to IGARCH estimates
    - High persistence and volatility indicates structural breaks
- State Space (SS) and Regime Switching (RS) models employed to assess this possibility
- These latter well specified models are compared on the basis of the efficiency criteria outlined above.
- This reveals a preference for the State Space models

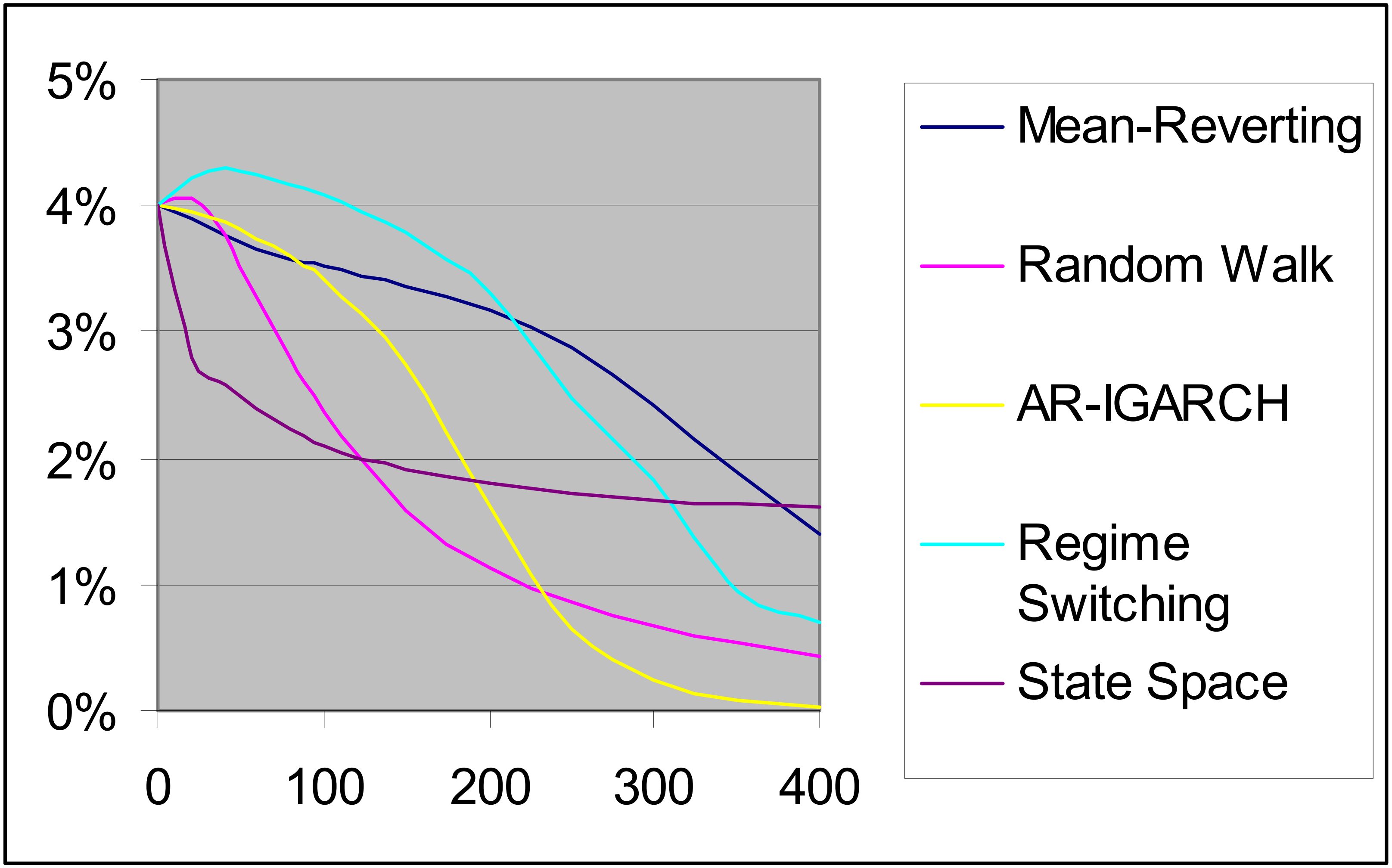
Numerical Simulations of Certainty Equivalent Discount Factors for the US

Year	4% Constant	Mean-Reverting	Random Walk	AR-IGARCH	Regime-Switching	State Space
1	0.96154	0.96154	0.96154	0.96154	0.96154	0.96154
20	0.45639	0.45906	0.45177	0.45876	0.45390	0.56424
40	0.20829	0.21661	0.20917	0.21250	0.19576	0.33136
60	0.09506	0.10471	0.10480	0.10062	0.08458	0.20296
80	0.04338	0.05150	0.05777	0.04894	0.03700	0.12889
100	0.01980	0.02567	0.03482	0.02455	0.01647	0.08408
150	0.00279	0.00476	0.01333	0.00529	0.00238	0.03132
200	0.00039	0.00095	0.00683	0.00178	0.00041	0.01255
250	0.00006	0.00022	0.00419	0.00104	0.00010	0.00526
300	0.00001	0.00006	0.00289	0.00086	0.00003	0.00227
350	0.00000	0.00002	0.00215	0.00080	0.00002	0.00100
400	0.00000	0.00001	0.00169	0.00078	0.00001	0.00044

Simulated Certainty Equivalent Rates for the US

Year	Mean-Reverting	Random Walk	AR-IGARCH	Regime Switching	State Space
1	4.00	4.00	4.00	4.00	4.00
20	3.91	4.05	3.96	4.22	2.79
40	3.76	3.76	3.88	4.31	2.59
60	3.65	3.28	3.74	4.26	2.38
80	3.58	2.80	3.60	4.18	2.23
100	3.51	2.37	3.42	4.09	2.10
150	3.36	1.59	2.75	3.79	1.91
200	3.16	1.14	1.62	3.31	1.79
250	2.87	0.85	0.65	2.46	1.72
300	2.43	0.66	0.23	1.83	1.67
350	1.87	0.53	0.09	0.95	1.64
400	1.41	0.44	0.04	0.70	1.61

Certainty Equivalent Discount Rates U.S.



8. Model Selection

- On the basis of the Coefficient of Variation and the proximity of 5% and 95% quantiles of the empirical distribution we cannot separate the SS and RS models.

- We evaluate the forecasting performance of models over a 30-year horizon using available real data (the annual nominal forward rates suggested by the term structure of the US government bonds).

- We calculate Mean Square Error (MSE) and compare four kernels, which weigh observations by their relevant proximity to the present.

Average MSE's for the US

Criterion	Mean-Reverting	Random Walk	AR-IGARCH	Regime-Switching	State Space
AMSE	2.058	2.171	2.102	2.323	1.832
AMSE (B)	1.692	1.724	1.692	1.687	1.499
AMSE (P)	1.725	1.746	1.720	1.683	1.426
AMSE (QS)	0.842	0.870	0.848	0.879	0.760
AMSE (TH)	1.769	1.797	1.765	1.738	1.550

9. Policy Implications of Model Selection

Value of Carbon emissions reduction: profile of benefits from Nordhaus DICE model

Value of Carbon (1989$/tC, Base Year 1995)

Model	Carbon Values ($/tc 400years)	Relative to Constant Rate	Relative to Mean Reverting	Relative to Random Walk
Regime Switch	5.22	-9.0%	-18.7%	-31.7%
Constant (4.0%)	5.74		-10.7%	-25.0%
IGARCH	6.37	+10.9%	-1.0%	-16.8%
N&P (MR)	6.43	+12.0%		-16.0%
N&P (RW)	7.65	+33.3%	+19.0%	
State Space	14.44	+151.7%	+124.7%	+88.8%

Implications for Nuclear Build in the U.K.

NPV of Nuclear Build with Carbon Credits using CERs

(£/kW)	CAPEX	OPEX	Decom	Rev's	Carbon Credits 400	NPV (Carbon Credits)
Constant (3.5%)	2172.8	2336.0	427.4	4062.4	227.9	-645.9
AR(4)	2167.0	2244.7	396.0	3903.7	215.4	-688.6
Regime Switch	2177.5	2401.0	479.4	4175.5	249.1	-633.4
State Space	2196.1	2973.1	1118.5	5170.4	547.3	-570.0

10. Conclusion

- Misspecification testing in both the US and UK real interest rate series lead us away from simple AR(p) towards time varying models, i.e. the State Space and Regime-Switching models.

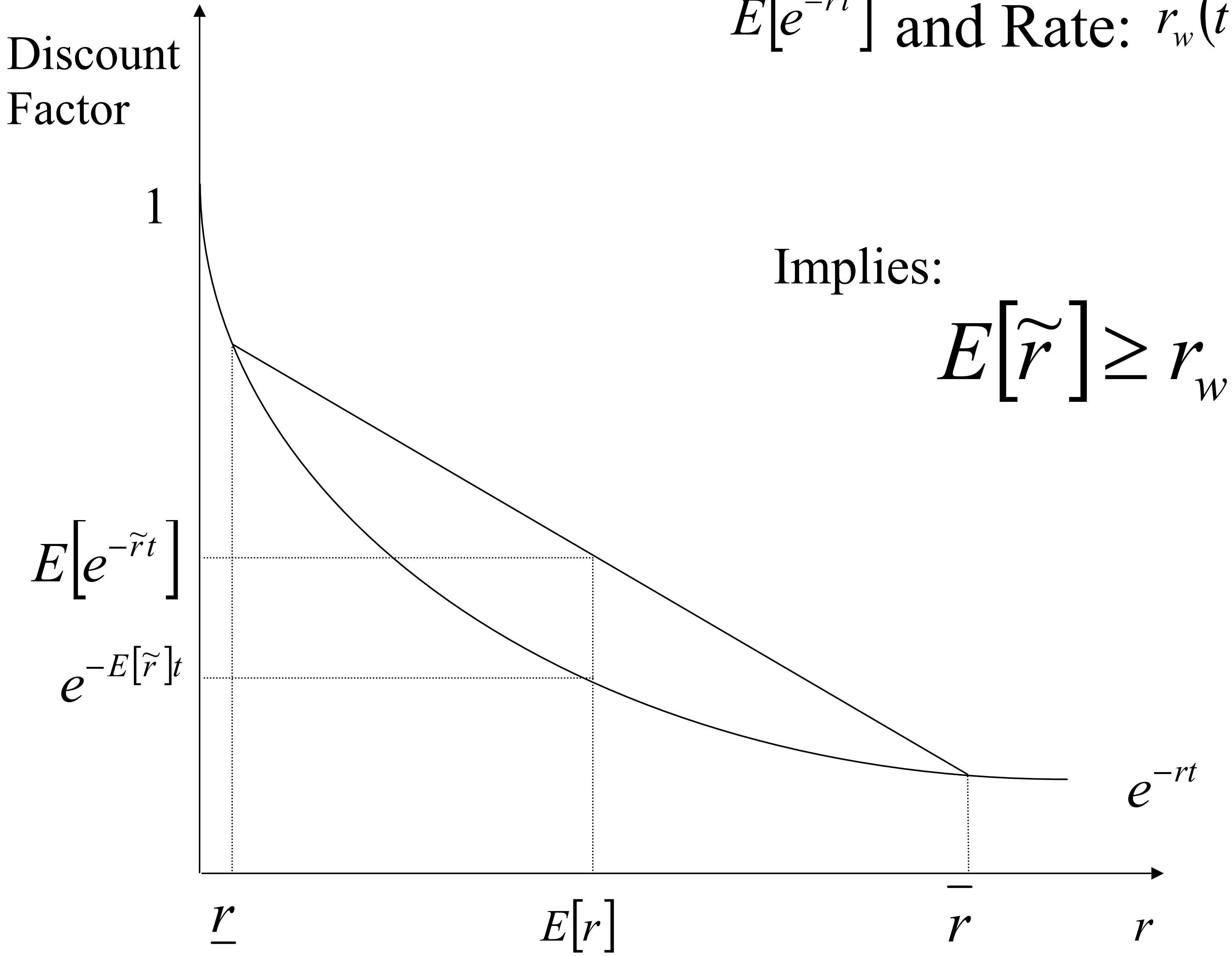
- As expected, the selection of the econometric model is of considerable moment in operationalising a theory of DDR's that depends upon uncertainty: econometric models contain different assumptions concerning the probability distribution and time series process of the object of interest.

  i) SS gives lower CER over the first 200 years, RS give the lower rate for period longer than 200 years.

  ii) RS and SS models give completely different measurements of the value of carbon reduction

- The out-of-sample forecast exercise reveals that the State Space model produces the more accurate forecasts and is preferred.

Certainty Equivalent Discount Factor $E\left[e^{-\tilde{r}t}\right]$ and Rate: $r_w(t)$



Implies:

$$E[\tilde{r}] \geq r_w(t)$$