Evaluating the carbon-macroeconomy relationship

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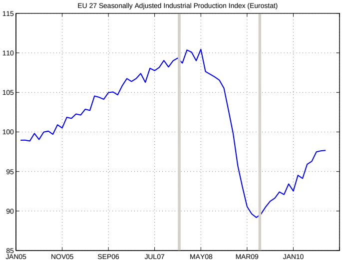
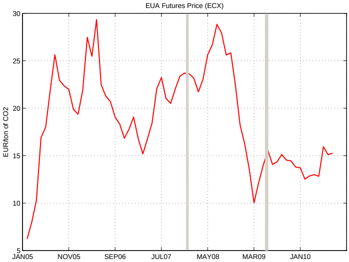
The Usual Suspects





- *What is the impact of economic activity on the growth rate of carbon prices from an empirical point of view?*

Outline

1. Literature review
2. Preliminary data analysis
3. Nonlinearity tests
4. Threshold autoregression and cointegration
5. Markov regime-switching VAR model

Related literature: Alberola et al. (2008, 2009)

$$
\begin{aligned}
p_t = {} & \alpha + \beta(L)p_t + \delta break + \nu psq_{i,t} + \varphi(L)ngas_t + \gamma(L)coal_t \\
& + \iota(L)elec_t + \kappa(L)dark_t + \lambda(L)spark_t + \sigma\ Win07 \\
& + \varsigma(L)cement_t + \tau(L)refin_t + \upsilon(L)coke_t + \omega(L)comb_t + \xi(L)glass_t \\
& + \psi(L)metal_t + \zeta(L)paper_t + \rho(L)ceram_t + \chi(L)iron_t + \epsilon_t
\end{aligned}
$$

$$
\begin{aligned}
p_t = {} & \alpha + \beta_i(L)p_t + \delta break_1 + \nu psq_{i,t} + \varphi(L)ngas_t + \gamma(L)coal_t \\
& + \iota(L)elec_t + \kappa(L)dark_t + \lambda(L)spark_t + \sigma\ Win07 \\
& + \omega\ sect_{i,j,t} + \emptyset\ sectpeak_{i,j,t} + \epsilon_t
\end{aligned}
$$

$$\sigma_t = \alpha_0 + \alpha^+(L)\epsilon_t^+ - \alpha^-(L)\epsilon_t^- + \beta(L)\sigma_t$$

Related literature: Chevallier (2009)

$$Y_t = \theta X_t' + \epsilon_t$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{k=1}^{r} \gamma_k \epsilon_{t-k}^2 \Gamma_{t-k}$$

- with $X_t' =$ $(Stock_t, Junk_t, TBill_t, Exc.Ret_t, Elec_t, Gas_t, Brent_t, D_{APR06}, D_{AUG07})'$

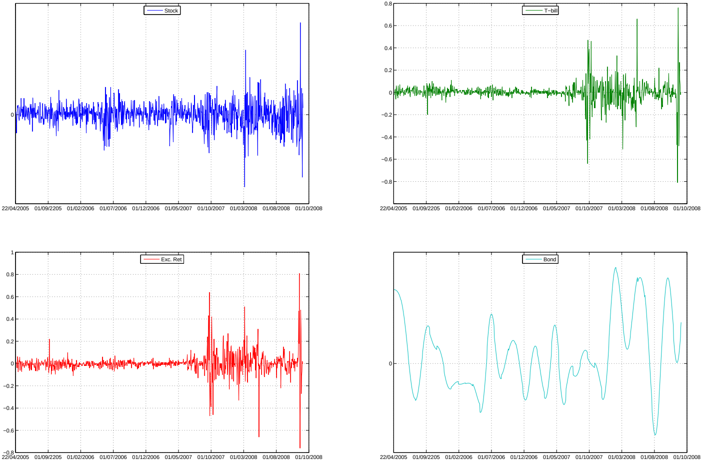


Figure: Dividend yield, junk bond yield, T-bill rate, excess return variables from April 22, 2005 to October 1, 2008

Source: Thomson Financial Datastream, U.S. Treasury, Reuters

Related literature: Chevallier (2011)

$$
\begin{aligned}
y_{it} =& \lambda_{0i} + \lambda_i f_t + \gamma_i r_t + \epsilon_{it} \\
f_t =& \Phi_1 f_{t-1} + \ldots + \Phi_p f_{t-p} + \epsilon_t^f
\end{aligned}
$$

$$
\begin{pmatrix} f_t \\ r_t \end{pmatrix} = \tilde{\Phi}_1 \begin{pmatrix} f_{t-1} \\ r_{t-1} \end{pmatrix} + \ldots + \tilde{\Phi}_p \begin{pmatrix} f_{t-p} \\ r_{t-p} \end{pmatrix} + \tilde{\epsilon}_t^f
$$

- with factors extracted from a broad dataset including macroeconomic, financial and commodities indicators.

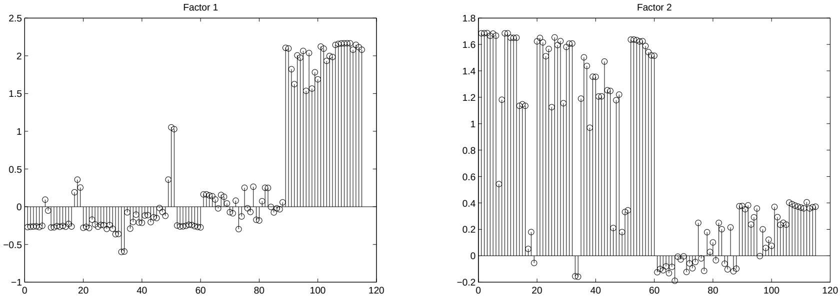


Figure: Factor loadings for data series in the FAVAR model

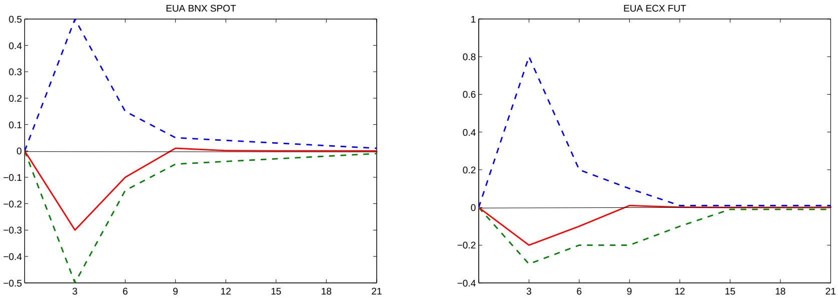
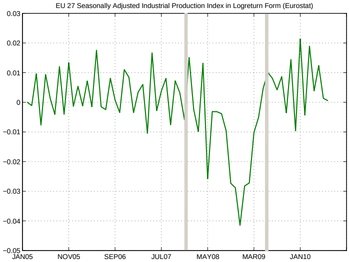


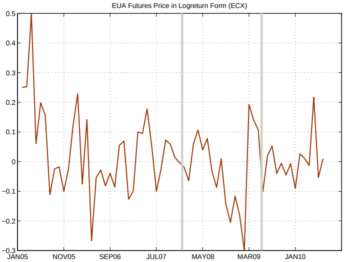
Figure: Impulse response functions of carbon prices following a global economic shock in the FAVAR model

Descriptive Statistics

	$EU27INDPROD$	$EUAFUT$
Mean	101.3036	18.92626
Median	102.1396	19.06000
Maximum	110.4501	29.33158
Minimum	89.18148	6.225364
Std. Dev.	6.269672	5.240281
Skewness	-0.387235	-0.091494
Kurtosis	1.960821	2.371070
JB	4.689154	1.197730
Prob. JB	0.095888	0.549435
LB test (p-value)	0.000001	0.000001
ARCH test (p-value)	0.002400	0.070500
Observations	67	67

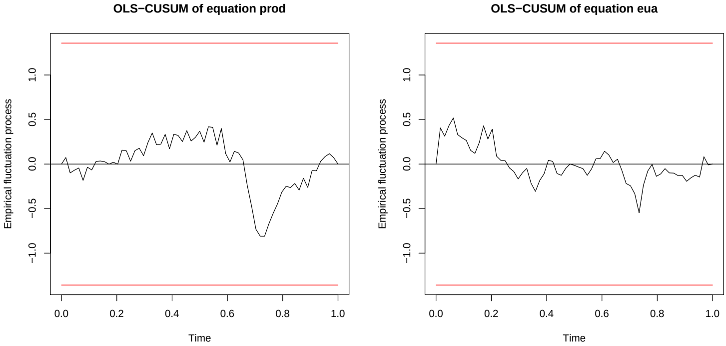
Logreturns





- there seems to remain some instability, especially for the EU 27 Industrial Production Index during May 2008 - March 2009

OLS-CUSUM Tests



- the empirical fluctuation processes do not seem to indicate the presence of structural breaks

ADF,PP,KPSS

	t-Statistic	Test critical values
ADF test statistic	-5.953323	-1.945903
PP test statistic	-5.892110	-1.945903

	LM-Stat.	Asymptotic critical values
KPSS test statistic	0.108755	0.146000

- linear unit root tests may not be adequate if the underlying process is nonlinear
- we also test explicitly for the presence of thresholds and unit root (Caner and Hansen (2001), Basci and Caner (2005))

Nonlinearity Tests: Results

Evidence of Nonlinearity	Keenan (1985)	Tsay (1986)	BDS (1987,1996)
EU27INDPRODRET	No	Yes	Yes
EUAFUTRET	No	No	Yes

Threshold Autoregression

SETAR$(2; p_1, p_2)$ model with delay d:

$$Y_t = \begin{cases} \phi_{1,0} + \phi_{1,1} Y_{t-1} + \ldots + \phi_{1,p_1} Y_{t-p_1} + \sigma_1 e_t, & \text{if} \quad Y_{t-d} \leq r \\ \phi_{2,0} + \phi_{2,1} Y_{t-1} + \ldots + \phi_{2,p_2} Y_{t-p_2} + \sigma_2 e_t, & \text{if} \quad Y_{t-d} > r \end{cases}$$

- ϕ's are autoregressive parameters, σ's are noise standard deviations, r is the threshold parameter, and $\{e_t\}$ is a sequence of i.i.d $\sim (0, \sigma^2)$ random variables.
- the autoregressive orders p_1 and p_2 of the two submodels need not be identical.
- the delay parameter d may be larger than the maximum autoregressive orders.

Testing for Threshold Nonlinearity

H_0: AR(p) model *vs.* H_1: two-regime SETAR model of order p with $\sigma_1 = \sigma_2 = \sigma$.

$$\begin{aligned} Y_t =& \phi_{1,0} + \phi_{1,1} Y_{t-1} + \ldots + \phi_{1,p} Y_{t-p} + \{\phi_{2,0} + \phi_{2,1} Y_{t-1} \\ & + \ldots + \phi_{2,p} Y_{t-p}\} \, I(Y_{t-d} > r) + \sigma e_t \end{aligned}$$

- $I(.)$ is an indicator variable that equals 1 if and only if the enclosed expression is true.
- $\phi_{2,0}$ represents the change in the intercept in the upper regime relative to that of the lower regime, and similarly interpreted are $\phi_{2,1}, \ldots, \phi_{2,p}$.
- $H_0 : \phi_{2,0} = \phi_{2,1} = \ldots = \phi_{2,p} = 0$.
- Under H_0, r is absent.
- Assuming $d \leq p$ and the validity of linearity, the large-sample distribution of the test does not depend on d.

Testing for Threshold Nonlinearity (ctd.)

LR test statistic:

$$T_n = (n-p)\log\left\{\frac{\hat{\sigma}^2(H_0)}{\hat{\sigma}^2(H_1)}\right\}$$

- $n-p$ is the effective sample size, $\hat{\sigma}^2(H_0)$ the maximum likelihood estimator of the noise variance from the linear AR(p) fit, and $\hat{\sigma}^2(H_1)$ from the SETAR fit with the threshold searched over some finite interval.
- Chan (1991) derived an approximation method for computing the p-values of the test.
- The test depends on the interval over which the threshold parameter is searched (from the $a \times 100^{th}$ percentile to the $b \times 100^{th}$ percentile of $\{Y_t\}$).
- choose a and b so that there are adequate data falling into each of the two regimes for fitting the linear submodels.

Testing for Threshold Nonlinearity: Results

for $EU27INDPRODRET$ with p=6

d	1	2	3	4	5
Test Statistic	46.570	43.209	64.870	25.273	58.466
p-value	0.000	0.000	0.000	0.012	0.000

for $EUAFUTRET$ with p=1

d	1	2	3	4	5
Test Statistic	1.133	1.597	4.494	3.037	7.250
p-value	0.116	0.311	0.341	0.297	0.192

Testing for Threshold Nonlinearity and Unit Root

- Caner and Hansen (2001), Basci and Caner (2005):

$$\begin{aligned}\Delta y_t =& \theta_1' x_{t-1} + e_t \quad \text{if} \quad |y_{t-1} - y_{t-m-1}| < \lambda \\ \Delta y_t =& \theta_2' x_{t-1} + e_t \quad \text{if} \quad |y_{t-1} - y_{t-m-1}| \geq \lambda\end{aligned}$$

- y_t is the selected time series, $x_{t-1} = (y_{t-1}, 1, \Delta y_{t-1}, \ldots, \Delta y_{t-k})'$ for $t = 1, 2, \ldots, T$, e_t is the i.i.d error term, m is the delay parameter and $1 \leq m \leq k$,
- The threshold variable is the absolute value of $y_{t-1} - y_{t-m-1}$,
- The threshold value λ is unknown and takes the value in the compact interval $\lambda \in \Lambda = [\lambda_1, \lambda_2]$ where these values are picked $P(|y_{t-1} - y_{t-m-1}| \leq \lambda_1) = 0.15, P(|y_{t-1} - y_{t-m-1}| \leq \lambda_2) = 0.85$.

Testing for Threshold Nonlinearity and Unit Root (ctd.)

- Decompose the coefficients:

$$\theta_1 = \begin{pmatrix} \rho_1 \\ \beta_1 \\ \alpha_1 \end{pmatrix}, \quad \theta_2 = \begin{pmatrix} \rho_2 \\ \beta_2 \\ \alpha_2 \end{pmatrix}$$

- where ρ_1 and ρ_2 are scalar, β_1 and β_2 have the same dimension as y_t, α_1 and α_2 are $k \times 1$ vectors.
- (ρ_1, ρ_2) represent the slope coefficients on y_{t-1}, (β_1, β_2) are the slopes on the deterministic components, and (α_1, α_2) are the slope coefficients on $(\Delta y_{t-1}, \ldots, \Delta y_{t-k})$ in the two regimes.
- Re-write the TAR model:

$$\Delta y_t = \theta_1' x_{t-1} 1_{\{|y_{t-1} - y_{t-m-1}| < \lambda\}} + \theta_2' x_{t-1} 1_{\{|y_{t-1} - y_{t-m-1}| \geq \lambda\}} + e_t$$

- with $1_{\{.\}}$ the indicator function.

Testing for Threshold Nonlinearity and Unit Root (ctd.)

- Under H_0(unit root) : $\rho_1 = \rho_2 = 0$

1. if the time series follows a stationary threshold autoregressive pattern, the alternative of interest is $H_1 : \rho_1 < 0, \rho_2 < 0$.
2. there is the case of partial unit root:

$$H_2 : \begin{cases} \rho_1 < 0 & \text{and} & \rho_2 = 0 \\ \rho_1 = 0 & \text{and} & \rho_2 < 0 \end{cases}$$

- If H_2 holds, then the time series is nonstationary, but we do not deal with a classic unit root.
- The test statistics for testing H_0 *vs.* H_1 and H_0 *vs.* H_2 are given by Caner and Hansen (2001). We test $H_0 : \rho_1 = \rho_2 = 0$ with a simple one-sided Wald as test statistic:

$$R_{1T} = t_1^2 1_{\{\hat{\rho}_1 < 0\}} + t_2^2 1_{\{\hat{\rho}_2 < 0\}}$$

- with t_1, t_2 the t-ratios for respectively $\hat{\rho}_1$ and $\hat{\rho}_2$. In order to test H_0 *vs.* H_2, we use the negative of the the t statistics $-t_1$,$-t_2$.

Bootstrap p-values of threshold unit root tests

Variable	R_{1T}	t_1	t_2
$EU27INDPROD$	0.002*	0.274	0.011*
$EUAFUT$	0.002*	0.014*	0.322

- Both R_{1T} statistics are significant at the 10% level.
- The one-sided Wald test (unit root *vs.* two-regime stationary nonlinear model) is rejected for both time series.
- $EU27INDPROD$: the rejection is due to the second regime, where the p-value for the t test is statistically significant.
- $EUAFUT$: the rejection is due to the first regime, where the p-value for the t test is statistically significant.
- $\rho_1 = 0$ and $\rho_2 < 0$ for $EU27INDPROD$.
- $\rho_1 < 0$ and $\rho_2 = 0$ for $EUAFUT$.
- $\rightarrow$ Both time series are partially stationary threshold processes.

AIC of the SETAR Models

for $EU27INDPRODRET$

d	AIC	$\hat{r}$	$\hat{p}_1$	$\hat{p}_2$
1	-405.80	-0.004	6	5
2	-409.30	-0.004	6	3
3	-427.70	-0.002	4	1
4	-399.60	-0.004	6	3
5	-409.60	-0.001	6	3

for $EUAFUTRET$

d	AIC	$\hat{r}$	$\hat{p}_1$	$\hat{p}_2$
1	-102.50	0.052	3	3
2	-97.79	0.011	0	0
3	-106.00	0.059	1	4
4	-101.40	0.011	1	2
5	-100.20	-0.007	0	1

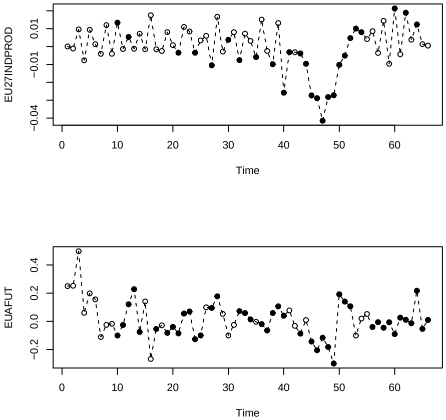
SETAR(2, 4, 1) Model with d=3 for $EU27INDPRODRET$

	Estimate	Std. Error	t-statistic	p-value
$\hat{d}$	3			
$\hat{r}$	-0.002			
Lower Regime (n_1 = 27)				
$\hat{\phi}_{1,0}$	-0.013	0.003	-4.688	0.001
$\hat{\phi}_{1,1}$	0.263	0.150	1.755	0.093
$\hat{\phi}_{1,2}$	1.098	0.161	6.842	0.001
$\hat{\phi}_{1,3}$	-0.808	0.222	-3.642	0.001
$\hat{\phi}_{1,4}$	-0.358	0.138	-2.605	0.016
$\tilde{\sigma}_1^2$	0.008			
Upper Regime (n_2 = 35)				
$\hat{\phi}_{2,0}$	0.007	0.001	6.878	0.001
$\hat{\phi}_{2,1}$	-0.545	0.967	-5.636	0.001
$\tilde{\sigma}_2^2$	0.005			

SETAR(2, 1, 4) Model with d=3 for *EUAFUTRET*

	Estimate	Std. Error	t-statistic	p-value
$\hat{d}$	3			
$\hat{r}$	0.059			
Lower Regime (n_1 = 43)				
$\hat{\phi}_{1,0}$	-0.025	-0.003	0.017	0.884
$\hat{\phi}_{1,1}$	0.306	0.306	0.0147	0.043
$\tilde{\sigma}_1^2$	0.109			
Upper Regime (n_1 = 17)				
$\hat{\phi}_{2,0}$	0.134	0.043	3.121	0.009
$\hat{\phi}_{2,1}$	-0.535	0.164	-3.271	0.007
$\hat{\phi}_{2,2}$	0.077	0.119	0.646	0.530
$\hat{\phi}_{2,3}$	-1.113	0.310	-3.595	0.004
$\hat{\phi}_{2,4}$	-0.187	0.096	-1.946	0.076
$\tilde{\sigma}_2^2$	0.063			

Thresholds Estimated by the SETAR Models



Note: Solid (open) circles indicate data in the lower (upper) regime of a fitted threshold autoregressive model.

- *EU27INDPROD*: data falling in the lower regime (with lag 3 values $<$ -0.002) are displayed as solid circles.
- The estimated lower regime corresponds mainly to the recession period.
- The estimated upper regime corresponds mainly to periods before/after the recession.
- *EUAFUT*: data falling in the lower regime (with lag 3 values $<$ 0.059) are displayed as solid circles.
- The estimated lower regime corresponds to periods of decreasing prices.
- The estimated upper regime corresponds mainly to periods of increasing prices.
- $\rightarrow$ From January 2009 onwards the carbon market is characterized by a lower regime, indicating a **delayed** adjustment to the financial crisis.

Model Diagnostics

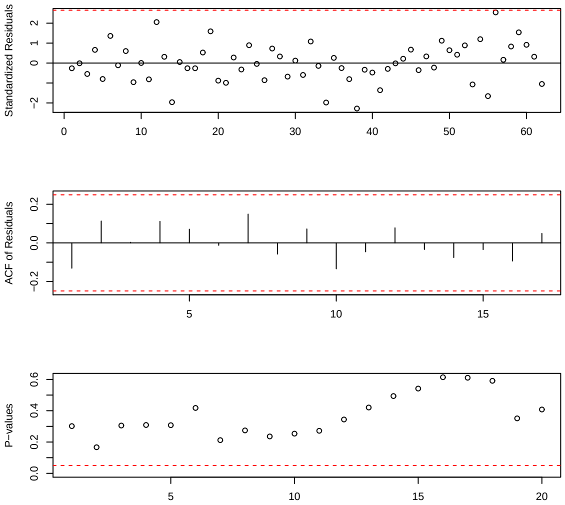
The dependence of the residuals necessitates the employment of a quadratic form of the residual autocorrelations:

$$B_m = n_{\text{eff}} \sum_{i=1}^{m} \sum_{j=1}^{m} q_{i,j} \hat{\rho}_i \hat{\rho}_j$$

- $n_{\text{eff}} = n - \max(p_1, p_2, d)$ is the effective sample size, $\hat{\rho}_i$ is the ith lag sample autocorrelation of the standardized residuals, and $q_{i,j}$ model-dependent constants
- If the true model is a SETAR model, $\hat{\rho}_i$ are likely close to zero and so is B_m, but B_m tends to be large if the model specification is incorrect.
- The quadratic form is designed so that B_m is approximately distributed as χ^2 with m degrees of freedom.

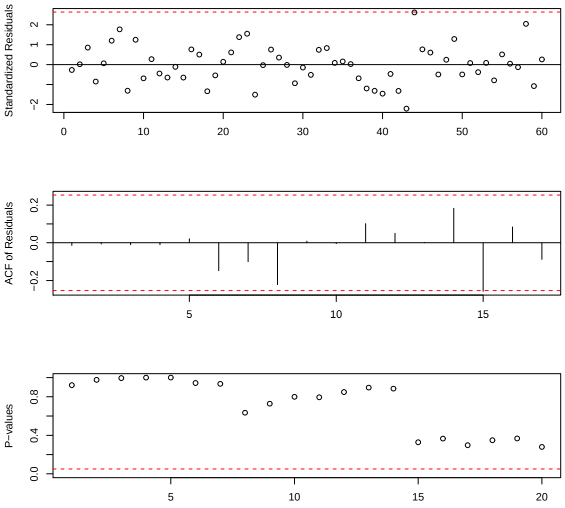
Model Diagnostics: Results

SETAR$(2,4,1)$ Model with d=3 for $EU27INDPRODRET$



Model Diagnostics: Results (ctd.)

SETAR$(2, 1, 4)$ Model with d=3 for $EUAFUTRET$



Linear Johansen Cointegration Rank Tests

for $LOG(EU27INDPROD)$ and $LOG(EUAFUT)$

Lags interval	(in first differences):	1 to 2		
Trace Test				
Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None*	0.3167	29.1068	18.3977	0.0011
At most 1*	0.0712	4.7299	3.8414	0.0296
Max-Eigen Test				
Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None*	0.3167	24.3769	17.1476	0.0037
At most 1*	0.0712	4.7299	3.8414	0.0296

Linear VECM Estimates

	$\Delta EU27INDPROD_t$	$\Delta EUAFUT_t$
Cointegrating Vector	0.1114	0.0110
μ	-0.1483***	0.0525
	(0.0326)	(0.0326)
w_{t-1}	1.1961***	-0.6181
	(0.2753)	(0.2753)
$\Delta EU27INDPROD_{t-1}$	0.1601***	0.0434
	(0.0545)	(0.0545)
$\Delta EUAFUT_{t-1}$	-0.0049	-0.0029
	(0.0135)	(0.0135)

$$\begin{pmatrix} \Delta EU27INDPROD_t \\ \Delta EUAFUT_t \end{pmatrix} = \mu + \alpha w_{t-1} + \Gamma \begin{pmatrix} \Delta EU27INDPROD_{t-1} \\ \Delta EUAFUT_{t-1} \end{pmatrix} + u_t$$

with $w_{t-1} = EU27INDPROD_{t-1} - \beta EUAFUT_{t-1}$.

Two-Regime Threshold Cointegration

Let x_t be a p-dimensional $I(1)$ time series which is cointegrated with one $p \times 1$ cointegrating vector β. Let $w_t(\beta) = \beta' x_t$ denote the $I(0)$ error-correction term. Following Hansen and Seo (2002):

$$\Delta x_t = \begin{cases} A_1' X_{t-1}(\beta) + u_t, & \text{if} \quad w_{t-1}(\beta) \leq \gamma \\ A_2' X_{t-1}(\beta) + u_t, & \text{if} \quad w_{t-1}(\beta) > \gamma \end{cases}$$

- A_1 and A_2 are the coefficient matrices governing the dynamics of the regimes.
- The error u_t is assumed to be a vector martingale difference sequence with finite covariance matrix $\sum = E(u_t u_t')$.
- The notation $w_{t-1}(\beta)$ and $X_{t-1}(\beta)$ indicate that the variables are evaluated at generic values of β.

Two-Regime Threshold Cointegration (ctd.)

All coefficients (except β) switch between the two regimes. The threshold effect only has content if $0 < \mathrm{P}(w_{t-1} \leq \gamma) < 1$, otherwise the model simplifies to linear cointegration. Therefore, we assume that:

$$\pi_0 \leq \mathrm{P}(w_{t-1} \leq \gamma) \leq 1 - \pi_0$$

- $\pi_0 > 0$ is a trimming parameter set to $\pi_0 = 0.05$ (see Andrews (1993), Andrews and Ploberger (1994)).
- The model is estimated by maximum likelihood under the assumption that the errors u_t are i.i.d Gaussian (algorithm developed by Hansen and Seo (2002)).
- Let the estimates be denoted by $(\tilde{\beta}, \tilde{A}_i, \tilde{\sum})$ with $\tilde{u}_t$ the residual vectors.

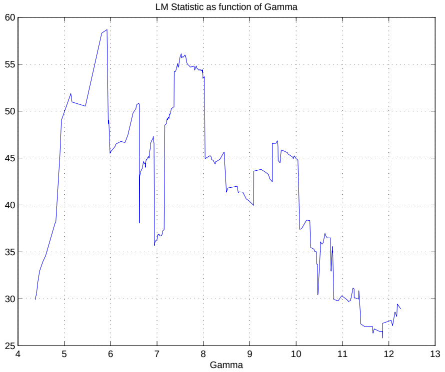
Testing for Threshold Cointegration

To test linear *vs.* threshold cointegration, Hansen and Seo (2002) suggest to use the LM test statistic proposed by Davies (1987):

$$\text{SupLM} = \sup_{\gamma_L \leq \gamma \leq \gamma_U} \text{LM}(\tilde{\beta}, \gamma)$$

- $[\gamma_L, \gamma_U]$ is the search region, so that γ_L is the π_0 percentile of $\tilde{w}_{t-1}$ and γ_U is the $(1 - \pi_0)$ percentile.
- As the function $\text{LM}(\tilde{\beta}, \gamma)$ is non-differentiable in γ, it is necessary to perform a grid evaluation over $[\gamma_L, \gamma_U]$.
- The LM statistics are computed with heteroskedasticity consistent covariance matrix estimates.

LM Statistic for the Two-Regime Threshold Cointegration Model



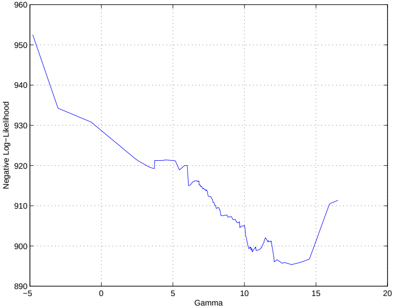
- SupLM test (estimated β) with 300 gridpoints
- the p-values are calculated by the parametric bootstrap

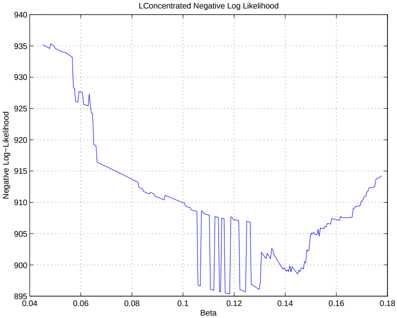
LM Tests Results for Threshold Cointegration

Lagrange Multiplier Threshold Test Statistic	58.6848
(Asymptotic) .05 Critical Value	19.5843
Bootstrap .05 Critical Value	19.7316
(Asymptotic) p-Value	0.0000
Bootstrap p-Value	0.0000

- all p-values were computed with 5,000 simulation replications.

Concentrated Negative Log-Likelihood of the Two-Regime Threshold Cointegrated Model





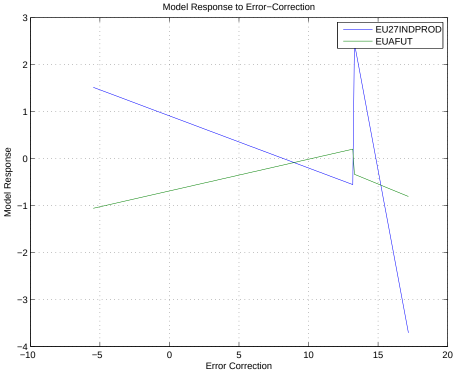
Threshold VECM Estimates

Threshold Estimate	13.2923	
Cointegrating Vector Estimate	0.1182	
First Regime	$\Delta EU27INDPROD_t$	$\Delta EUAFUT_t$
μ	-0.1109***	0.0674
	(0.0317)	(0.0700)
w_{t-1}	-0.9083***	0.6894
	(0.2718)	(0.7057)
$\Delta EU27INDPROD_{t-1}$	0.3033***	0.0409
	(0.0594)	(0.0395)
$\Delta EUAFUT_{t-1}$	-0.0219	-0.0004
	(0.0116)	(0.0183)
Percentage of Observations	0.9415	
Second Regime	$\Delta EU27INDPROD_t$	$\Delta EUAFUT_t$
μ	-1.5883***	-0.1215
	(0.2897)	(0.2339)
w_{t-1}	-23.5795***	-1.2808
	(4.2598)	(3.2803)
$\Delta EU27INDPROD_{t-1}$	-0.5070***	0.2195***
	(0.1453)	(0.0806)
$\Delta EUAFUT_{t-1}$	0.6383	-0.5704**
	(0.1125)	(0.2155)
Percentage of Observations	0.0584	

- The estimated threshold is $\hat{\gamma} = 13.292$ with the error-correction term defined as $w_t = EU27INDPROD_t - 0.118EUAFUT_t$.
- 'Typical' regime:
  $EU27INDPROD_t \leq 0.118EUAFUT_t + 13.292$, *i.e.* when the EU industrial production index is less than 13.3 percentage points above the carbon futures price. (94% obs.)
- 'Extreme' regime:
  $EU27INDPROD_t > 0.118EUAFUT_t + 13.292$, *i.e.* when the gap is above 13.3%. (6% obs.)
- The EU industrial production index impacts positively the EUA futures price in regime 2 at the 1% significance level.
- $\rightarrow$ the carbon-macroeconomy relationship goes from the EU industrial production index (lagged one period) to the carbon futures price, with a coefficient equal to 0.22.
- The EUA futures price has no statistically significant effect on the EU industrial production index in either of the regimes.

- $EU27INDPROD$ governs most of the adjustment from the short-run to the long-run equilibrium of the model: its coefficients for w_{t-1} are highly significant in both regimes.
- the magnitude of the response for $EU27INDPROD$ is between 1.33 (regime 1) and 18 (regime 2) times greater than the coefficient of $EUAFUT$.
- $EUAFUT$: the error-correction term is not statistically significant.
- The 4 error-correction coefficients (for EU industrial production and carbon futures prices in both regimes) are either negative or insignificantly different from zero if positive.

Model Response to the Error Correction



- This figure plots the error-correction effect, *i.e.* the estimated regression functions of $\Delta EU27INDPROD_t$ and $\Delta EUAFUT_t$ as a function of w_{t-1} by holding other variables constant.

- RHS of the threshold: strong error-correction effect for $EU27INDPROD$ and minimal error-correction effect for $EUAFUT$.
- LHS of the threshold: minimal error-correction effect for $EU27INDPROD$ and flat near-zero error-correction effect from $EUAFUT$.
- **Asymmetry** for $EU27INDPROD$: there is a stronger error-correction effect in the 'extreme' regime.
- The w_{t-1} coefficient for $EU27INDPROD$ is especially important (-23.58) in the 'extreme' regime (at 1% level), which implies a mean-reverting dynamic behavior of the 'gap' between the two time series once the threshold (13.3 percentage points) has been reached.
- A value of the gap above 13.3 percentage points in one month will produce a *downward* pressure on the industrial production index in the subsequent month in order to restore the long-run equilibrium relationship.
- → Industrial production leads the nonlinear mean-reverting behavior of the carbon price, but not vice versa.

Markov-Switching VAR

Consider an n-dimensional vector $\boldsymbol{y}_t \equiv (y_{1t}, \ldots, y_{nt})'$ which is assumed to follow a VAR(p) with parameters:

$$\boldsymbol{y}_t = \boldsymbol{\mu}(s_t) + \sum_{i=1}^{p} \boldsymbol{\Phi}_i(s_t) y_{t-i} + \epsilon_t$$
$$\boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}(s_t))$$

- The parameters for the conditional expectation $\boldsymbol{\mu}(s_t)$ and $\boldsymbol{\Phi}_i(s_t)$, $i = 1, \ldots, p$, as well as the variances and covariances of the error terms $\boldsymbol{\epsilon}_t$ in the matrix $\boldsymbol{\Sigma}(s_t)$ all depend upon the state variable s_t which can assume a number q of values (corresponding to different regimes).

Markov-Switching VAR (ctd.)

- The general idea behind the class of Markov-switching models is that the parameters and the variance of an autoregressive process depend upon an *unobservable* regime variable $s_t \in \{1, \ldots, M\}$, which represents the probability of being in a particular state of the world.
- A complete description of the Markov-switching model requires the formulation of a mechanism that governs the evolution of the stochastic and unobservable regimes on which the parameters of the autoregression depend.
- Once a law has been specified for the states s_t, the evolution of regimes can be inferred from the data.

Markov-Switching VAR (ctd.)

Typically, the regime-generating process is an ergodic Markov chain with a finite number of states defined by the transition probabilities:

$$p_{ij} = Prob(s_{t+1} = j | s_t = i), \quad \sum_{j=1}^{M} p_{ij} = 1 \quad \forall i, j \in \{1, \dots, M\}$$

- The transition probabilities of the Markov-process determines the probability that volatility will switch to another regime, and thus the expected duration of each regime.

Markov-Switching VAR: Results (1/2)

Log-likelihood	270.01
μ (Regime 1)	0.0014***
	(0.0009)
μ (Regime 2)	-0.0048***
	(0.0006)

Equation for $EU27INDPRODRET$	$EU27INDPRODRET$	$EUAFUTRET$
ϕ_1 (Regime 1)	0.1387*	-0.0003
	(0.0777)	(0.0043)
ϕ_1 (Regime 2)	-0.2079	0.0192
	(0.3383)	(0.0213)
ϕ_2 (Regime 1)	0.4343***	0.0043
	(0.1178)	(0.0136)
ϕ_2 (Regime 2)	1.3421***	0.0134
	(0.5488)	(0.0559)

- Regime 1 (expansion): output growth per month is equal to 0.14% on average.
- Regime 2 (recession): the average growth rate amounts to -0.48%.
- $\rightarrow$ the effects of the recessionary shock are found to be quite strong.
- AR(2) seems to describe the autocorrelation structure of $EU27INDPRODRET$.
- No statistically significant impact of carbon futures on EU industrial production.

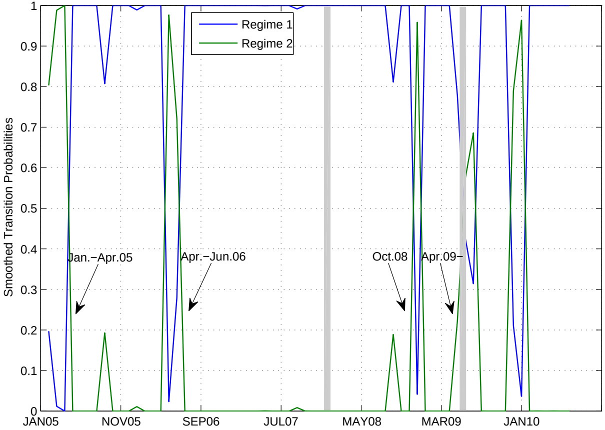
Markov-Switching VAR: Results (2/2)

Equation for $EUAFUTRET$	$EU27INDPRODRET$	$EUAFUTRET$
ϕ_1 (Regime 1)	1.3369	0.1253*
	(1.1483)	(0.0760)
ϕ_1 (Regime 2)	-1.5452	0.3455
	(8.7970)	(0.5497)
ϕ_2 (Regime 1)	0.7754***	-0.0313**
	(0.1893)	(0.0151)
ϕ_2 (Regime 2)	-0.9257***	1.7236
	(0.2448)	(1.4546)
Standard error (Regime 1)	0.0009	
Standard error (Regime 2)	0.0013	
Transition Probabilities Matrix	**Regime 1**	**Regime 2**
Regime 1	0.8873***	0.5189*
	(0.1700)	(0.3111)
Regime 2	0.1211	0.4927
	(0.0815)	(0.4326)
Regime Properties	**Prob.**	**Duration**
Regime 1	0.8029	8.60
Regime 2	0.1971	1.96

- For $EUAFUTRET$, the process seems characterized by an AR(1).
- EU industrial production (variable $EU27INDPRODRET$) has two kinds of delayed impacts on carbon futures:
- *positive* during Regime 1 (as ϕ_2 is equal to 0.78 and highly significant),
- *negative* during Regime 2 (as ϕ_2 is equal to -0.93 and highly significant).
- $\rightarrow$ **delayed** impact of macroeconomic activity on carbon markets.

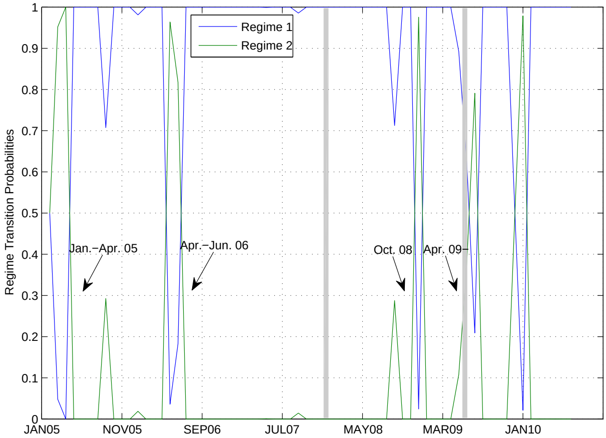
- During an expansionary phase, the series are most likely to remain in Regime 1 (88.73%).
- The probability that the series switch from Regime 1 to Regime 2 is equal to 12.11%.
- Once the economy finds itself in a depression, the probability that it will be in a depression the following month is 49.27%.
- If the economy is in the recessionary phase, the probability that it will change directly to a growth regime is equal to 51.89%.
- Regime 1 is assumed to last 8.60 months on average.
- Regime 2 is assumed to last 1.96 months on average.
- The economy would spend about 80% of the time spanned by our sample of data in regime 1.
- Regime 2 has an ergodic probability of about 20%.

Smoothed Transition Probabilities



- the smooth probability, which is the probability of a particular state in operation at time t conditional on all information in the sample.

Regime Transition Probabilities



- the regime probability at time t is the probability that state t will operate at t, conditional on information available up to $t-1$.

Model Diagnostics

	Markov-switching VAR	
LR Statistic	18.226	
p-value	0.001	
Symmetry test	1.698	
p-value	0.047	

Distributional Characteristics	*EU27INDPRODRET*	*EUAFUTRET*
Mean	-0.0001	0.0081
Median	0.0002	-0.0126
Maximum	0.0156	0.3122
Minimum	-0.0237	-0.2777
Std. Dev.	0.0098	0.1236
Skewness	-0.5717	0.1229
Kurtosis	2.7062	2.8208

- The carbon-macroeconomy relationship is better described by a two-regime Markov-switching model than by the random walk model.
- We reject the hypothesis of symmetry of the Markov transition matrix (which implies symmetry of the unconditional distribution of the growth rates) at the 5% level.
- The Markov-switching model produces both the degree of skewness and the amount of kurtosis that are present in the original data.

Key Messages

1. macroeconomic activity is likely to affect carbon prices with a lag, due to the specific institutional constraints of this environmental market
2. the carbon-macroeconomy relationship seems adequately captured by two-regime threshold error-correction and two-regime Markov-switching VAR models compared to linear models as main competitors

Further Work

- Dynamic correlations between industrial production and carbon prices in the DCC-MIDAS model (Colacito et al. (2010), Baele et al. (2010))
- Smooth transition error-correction models (Martens et al. (1998), (1998))
- Markov error-correction models (Psaradakis et al. (2004))
- Time-varying transition-probability Markov-Switching models (Filardo (1994))
- Markov-Switching GARCH models (Marcucci (2005), Henry (2009), Janczura and Weron (2010))

Thanks for your attention!

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